REMARKS

Reconsideration of the application, as amended, is respectfully requested.

Claims 16, 21, 22 and 23 are cancelled above without prejudice.

Rauwendal, '159 concerns screw extruders with improved dispersive mixing. It is apparent that the material to be mixed must be a fluid of high viscosity:

"the material is conveyed . . . through a melting zone where the material is heated under carefully controlled conditions to melt the material" (column 1, lines 1-2)

"in screw extruders, significant mixing occurs only after the polymer has melted" (column 2, lines1-2)

"dispersive mixing should be done at as low a temperature as possible to increase the viscosity of the fluid, and with it the stresses in the polymer melt" (column 2, lines 9-12)

The Office asserts that the third statement shows that the extruder of '159 could be used for applications wherein cooling is required. However, when read in the context of the preceding statement of a heating step, it is clear that this does not refer to the application of cooling. In fact it refers to the necessity to provide the correct amount of heating, i.e. sufficient to melt the material but not to raise its temperature too much so that its viscosity decreases. '159 obtain the correct viscosity for optimal mixing. Thus Applicants submit that the Office has not pointed to any teaching in '159 that

Rauwendal's extruder could or should be used for applications wherein cooling is required so that the product leaves the extruder colder than when it entered.

Furthermore, there is a fundamental difference between the flow in extruders in which the barrel is cooled with liquid ammonia and those in which the barrel is not cooled. In extruders such as those disclosed by '159, a combination of viscous dissipation and external heating warms the product so that it is a fluid. In particular, the product is a fluid close to the barrel wall. By contrast, in an extruder cooled with liquid ammonia the barrel wall is very cold, and the material close to the barrel freezes solid and sticks to it. This leads to an entirely different boundary condition to the flow. It is well known that boundary condition has a strong influence on the flow (see for example the paper, Abstract of "Slip flow in partially filled screw channel" J. Reinforced Plastics and Composites 17 (1998) 712 (Cheng et al.) Therefore, flow in identical extruders but with different boundary conditions is completely different. One of ordinary skill in the art would not attempt to apply the teaching with regard to the preferred geometry of the screw for an extruder in which the fluid is liquid at the wall to one in which the fluid solidifies at the wall.

At column 14, lines 43-45, '159 states that "it is possible to mix food products such as dough, mashed potatoes, cooking oil, a slurry of grapes or fruit concentrates, honey or peanut butter." This makes it clear that the teaching with regard to mixing of polymer melts can also be applied to the mixing of foods – provided that the food satisfied the requirement that it is a fluid of high viscosity. This is the case for all of the foods listed at or around room temperature. However, at the much lower temperatures produced by cooling with liquid ammonia, these foods would be solid, and therefore the extruder would not work. Thus, even if the person of ordinary skill in the art were to provide the extruder of '159 with cooling, he or she would not be motivated to use liquid ammonia.

'159 concerns mixing. The Office points to no teaching that the geometry of the extruder of '159 will be suitable for an extruder which is cooled with liquid ammonia. In particular, there is no teaching in '159 that the design will provide optimal cooling while maintaining flow.

Therefore, it is submitted that there is no reason to believe that any person skilled in the art would combine '159 and Fels. This combination constitutes an ex post facto analysis done with the benefit of hindsight knowledge of the present invention.

Moreover, even if a person skilled in the art were to attempt to combine '159 and Fels, '159 discloses a single screw extruder whereas Fels discloses a twin screw extruder. Twin screw extruders are completely different from single screw extruders (see the paper, "Engineering aspects of single and twin-screw extrusion-cooking of biopolymers" J. Food Eng. 2 (1983) 157-175, D.J. van Zuilichem and W. Stolp (van Zuilichem.) So when combining them, would a single screw or a twin screw be chosen?

Furthermore, even if such a combination were made, the result would be an extruder with a pitch angle outside the range claimed in the present application. The pitch angle range disclosed by '159 is broad; in fact at column 11, lines 2-3, it is stated that any angle in the range –90 to +90 degrees will work well. Fels specifies that the appropriate screw pitch angle for an extruder cooled with liquid ammonia is 20 to 30 degrees (column 7, lines 10-11). Thus, the person skilled in the art combining the teachings of '159 and Fels would choose the pitch angle taught by Fels for extruding ice cream from the choice allowed by '159. As stated in the present application at page 2, lines 23-28, Applicants have discovered that the performance of the extruder when used in the manufacture of ice cream can be dramatically improved when operating with a pitch angle outside those previously used. The data on page 8 of the present application shows that the ice cream temperature is significantly lower (note that even 1°C lower in

temperature provides a significant advantage) for the pitch angles of 35 and 40 degrees than for various pitch angles in the range 12 – 28 degrees.

In view of the foregoing, it is respectfully requested that the application, as amended, be allowed.

Respectfully submitted,

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Engineering Aspects of Single- and Twin-screw Extrusion-cooking of Biopolymers*

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ABSTRACT

A survey is given of the properties of single- and twin-screw extruders. The influence on the design of the different leakage gaps existing in co-rotating, counter-rotating, self-wiping, twin-screw extruders and single-screw equipment is discussed. The mixing effects in single- and twin-screw equipment and the shear distribution and shear levels that can be generated in the equipment are discussed. The overheating effect possible in single-screw extruders is related to the type of flow in the extruder channel. Finally, the properties and power consumption of Cincinnati conical, twin-screw extruders are discussed.

INTRODUCTION

When considering the cooking-extrusion of foods and feeds, a distinction can be made between single-screw extruders (s.s.e.'s) and twin-screw extruders (t.s.e.'s), the main difference being in the conveying mechanism (see Table 1). In the s.s.e. the conveying action is the result of two friction effects: first the friction between screw and product and second the friction between barrel and product. The s.s.e. *needs* the barrel wall for a good conveying action and the barrel wall will be an

*Presented at the EXTRUSION COOKING Symposium, 7th World Cereal and Bread Congress, Prague, June/July, 1982.

TABLE 1
Main Differences Between Single- and Twin-screw Extruders

	Single-screw extruder	*Twin-screw extruder*
Main energy supply	Viscous dissipation	Heat transfer to barrel
Transport mechanism	Friction between metal and food material	Positive displacement
Throughput capacity	Dependent on moisture- and fat-content and pressure	Independent
Approximate specific power consumption per kg product	900–1 500 kJ kg^{-1}	400–600 kJ kg^{-1}
Heat distribution	Large temperature differences	Small temperature differences
Mechanical power dissipation	Large shear forces	Small shear forces
Degassing possibilities	Simple	Difficult
Rigidity	High	Bearing construction is vulnerable
Capital costs	Low	High
Minimum water content	10%	8%
Maximum water content	30%	95%

important part of the design. In a t.s.e. with closely intermeshing screws the product is enclosed between screws and barrel in C-shaped chambers (see Fig. 1), is therefore prevented from rotation with the screws, and so is conveyed positively towards the die. Here, depending on process conditions, friction at the barrel wall is less important. However, the screw geometry itself is important, as it is now possible to influence the pressure built up in the chambers and the resulting leakage flow back from one chamber to the previous one or across to the neighbouring screw.

An s.s.e. can be compared to and modelled as a continuous channel with a pseudo fully-developed flow profile and accompanying temperature profile. This is in contrast with the conditions in a closely-

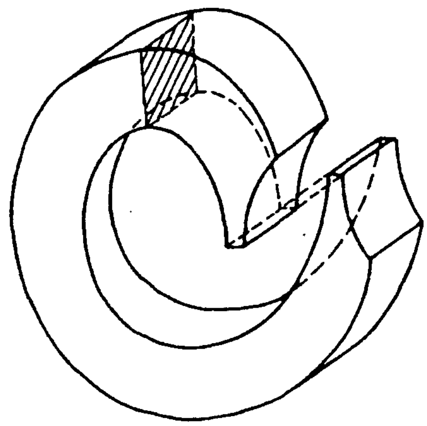

Fig. 1. C-shaped chamber of twin-screw extruder.

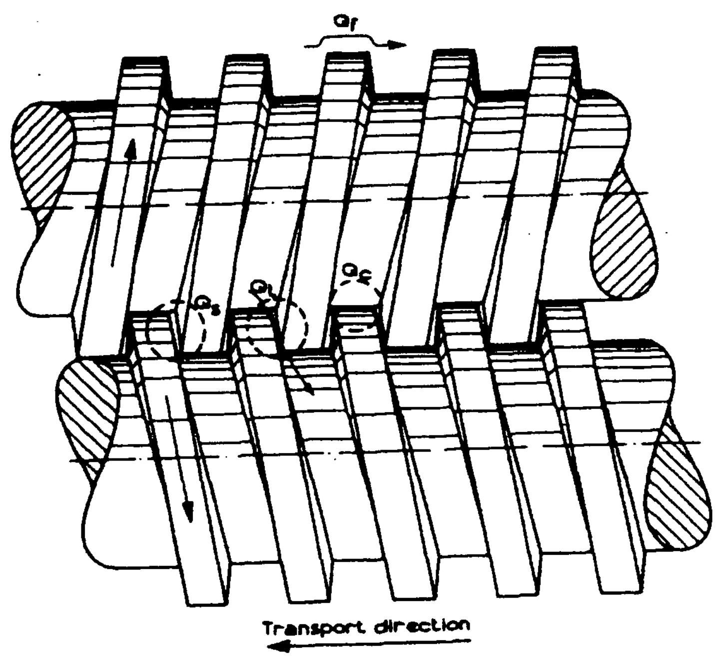

Fig. 2. Leakage flows in a closely-intermeshing, counter-rotating twin-screw extruder.

intermeshing t.s.e. forming closed C-shaped chambers so that no fully-developed profiles can be present. Design requirements and shear limitations mean that t.s.e.'s are never fully sealed, and that there is a certain interconnection between the chambers resulting in a certain leakage flow (see Fig. 2) which can be small, as in the small gaps of the pumping zone in counter-rotating extruders, or can be considerable in the wide gaps of a self-wiping co-rotating t.s.e. (see Figs 3, 4, 5 and 6). Such an extruder can also be described using the model of a pseudo-continuous channel with large ducts; so too can non-intermeshing screw combinations where clear continuous channels exist along the whole length of the extruder as with the s.s.e. There will remain the difference of incompletely-developed flow and temperature profiles.

If a classification of extruders is required, then it is logical to introduce a distinction based on the 'openness' of the channel, or the restrictions limiting continuous action. In intermeshing t.s.e.'s, the action is governed by positive displacement with leakage, the drag flow and the pressure flow being considered as secondary effects.

Self-wiping t.s.e.'s can be regarded as a special, intermediate, case. Although the intermeshing region forms a barrier for backflow and

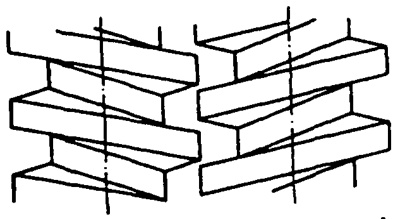

Fig. 3. Example of twin-screw extruder: non-intermeshing.

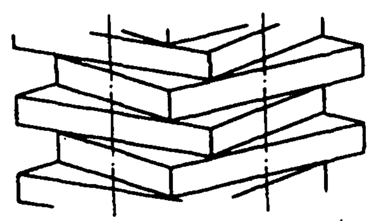

Fig. 4. Example of twin-screw extruder: intermeshing counter-rotating.

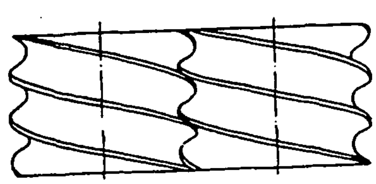

Fig. 5. Example of twin-screw extruder: self-wiping.

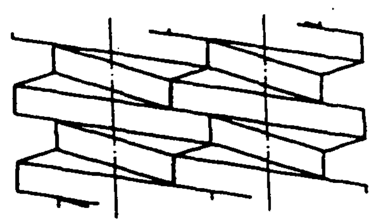

Fig. 6. Example of twin-screw extruder: intermeshing co-rotating.

produces a positive displacement action, because of the openness of the channel, the balance between drag flow and pressure flow largely determines the transport.

CO-ROTATING VERSUS COUNTER-ROTATING TWIN SCREW EXTRUDERS

Conclusion A: leakage gap design

Although the mechanism of transportation in co-rotating and counter-rotating, closely-intermeshing t.s.e.'s is similar, there are some differences in their flow fields and mixing effects. Pressure is built up in the tangential direction around the C-shaped chambers. With counter-rotating screws, there tends to be a build up of pressure on the converging side of the screws while on the diverging side there is a low pressure region (Figs 7 and 9(a)). This pressure difference in the screw chambers mainly influences the leakage through the calender gap in question and through the side gap. On the other hand, with co-rotating screws it can be concluded that the tangential pressure build up influences the leakage through the tetrahedron gap most (see Figs 7 and 9(b)). Further, the drag component of the leakage flows favours the calender gap and side gap most in counter-rotating machines, whereas in co-rotating extruders the resultant of the drag flow will be mainly through the tetrahedron gap. From geometrical considerations (the screws must fit into each other) the tetrahedron gap must be designed much bigger in size in co-rotating than in counter-rotating twins. This can be seen qualitatively in Fig. 8, where cross-sections in the mid-plane between and parallel to the screw axes are drawn for typical counter-rotating and co-rotating screws.

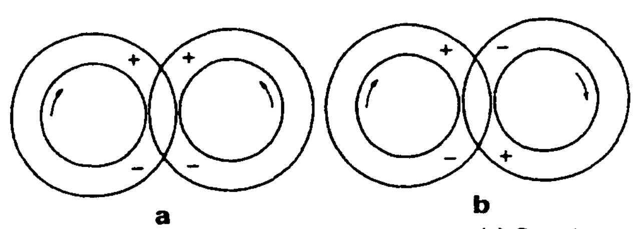

Fig. 7. Pressure distribution in twin-screw extruders. (a) Counter-rotating; (b) co-rotating.

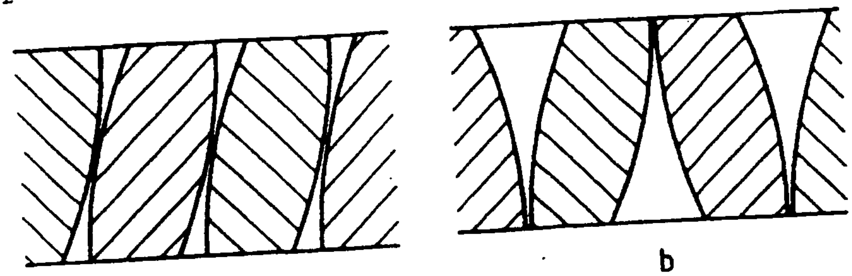

Fig. 8. Cross-section of the screws at the intermeshing part. (a) Counter-rotating; (b) co-rotating.

Conclusion B: mixing

All the above factors favour flow through the tetrahedron gap in co-rotating systems. Since the tetrahedron gap is the only gap that connects the chambers associated with one screw with those formed by the other, it is clear that in co-rotating extruders the material from the one screw mixes fairly well with that from the other screw. In counter-rotating systems, however, the mixing of the material carried by the two individual screws is much less, but the positive displacement action is greater. The self-wiping, co-rotating t.s.e., as shown in Fig. 5, forms an extreme case. Here all the gaps are minimised except for the tetrahedron gap. The material flows very easily from one screw to the other, thus ensuring good mixing between the material carried by the two screws – at the cost of positive displacement action. The only restriction to back flow is the 'kink' that is formed in the channel, no relevant intermeshing in the channel direction being present.

SHEAR DISTRIBUTION IN VARIOUS EXTRUDERS

Shear is an important consideration in the choice of an extruder. Some food materials have to be handled gently and low shear levels are required. Other materials may demand high shear for the whole processing period or need a high shear treatment for a short time. Depending on the application then, minimum, maximum or average shear rates may exist which are necessary for the achievement of particular process goals. The maximum shear rate in an extruder will usually be achieved

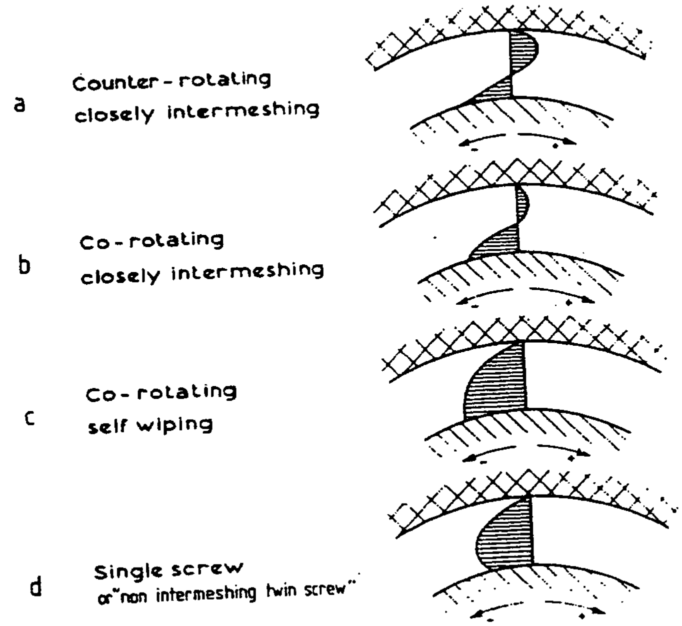

Fig. 9. Flow profiles in different extruder types.

in the flow through one of the leakage gaps: in a single-screw extruder this will of course be the flight gap; in twin-screw machines there are more gaps. In both cases, however, the quantity of material leaking through the flight gap is small, and the effects of this mechanical working on the bulk of the product is usually slight, except in so far as it may be important in the melting region.

In order to compare the average shear in various types of extruders, the positive displacement action should be considered. Figure 9(a) shows that closely-intermeshing, counter-rotating t.s.e.'s have virtually no net back leakage in the channel, which means that the positive displacement efficiency is very high. In closely-intermeshing, co-rotating t.s.e.'s (Fig. 9(b)) the leakage gaps are larger, thus diminishing the positive displacement action. This is even more so in self-wiping, co-rotating t.s.e.'s (Fig. 9(c)). In the case of s.s.e.'s (Fig. 9(d)) the positive conveying action with a given back pressure is very small because of the unrestricted continuous channel. Since, in general, the channel depth at comparable screw diameters also decreases when going from closely-

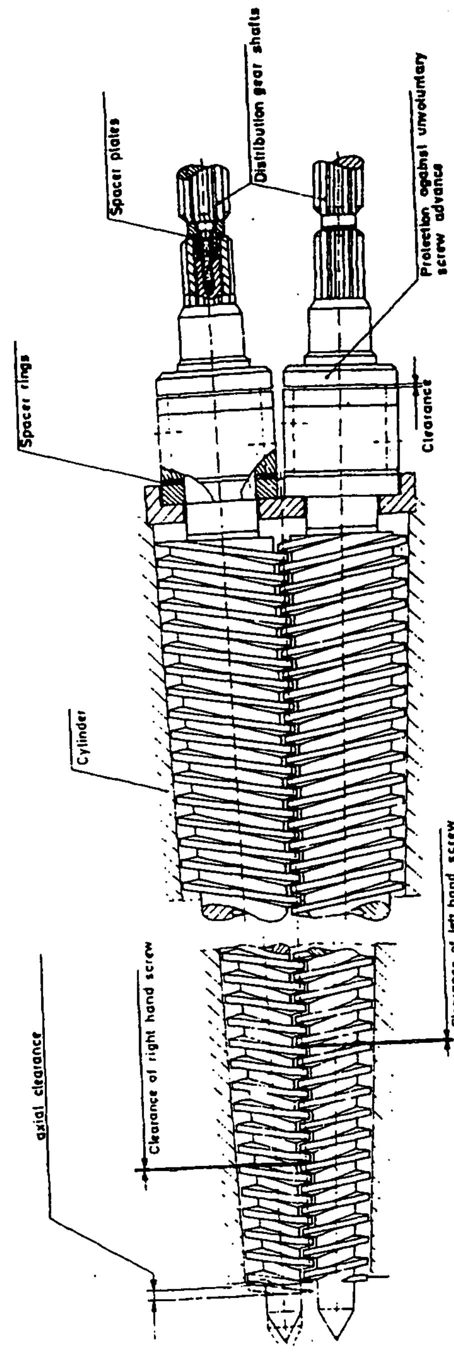

Fig. 10. Example of a screw-set for a conical counter-rotating twin-screw extruder (Cincinnati).

intermeshing to single-screw extruders it is obvious that for comparable output rates both single-screw extruders and self-wiping extruders must operate at much higher rotational speeds than closely-intermeshing, co-rotating and counter-rotating machines. Because of the wider clearances, which are related to the smaller positive displacement action, high rotation speeds can be more easily achieved in self-wiping, twin-screw and in single-screw extruders than in closely intermeshing twin-screw extruders. It can now be concluded from the relationship between rotational speed requirements and the relative channel and chamber depths that the average shear rates will be highest in single-screw extruders, and will be successively lower with the self-wiping, the closely-intermeshing, co-rotating and closely-intermeshing, counter-rotating geometries.

As stated above, the greatest shear levels in the extruder will occur in the various leakage gaps. In single-screw and self-wiping extruders all channels are large except for the flight gap and the maximum shear rate will be of the same magnitude as the average. This is not true for closely-intermeshing extruders. In co-rotating, closely-intermeshing machines a special shear region is present in the tetrahedron gap. The shear in this gap, however, is not very large since the walls of this gap move in essentially the same direction. Although an extremely high shear can be experienced in the calender gap this is not usually significant since virtually no material will be transported through this gap (see

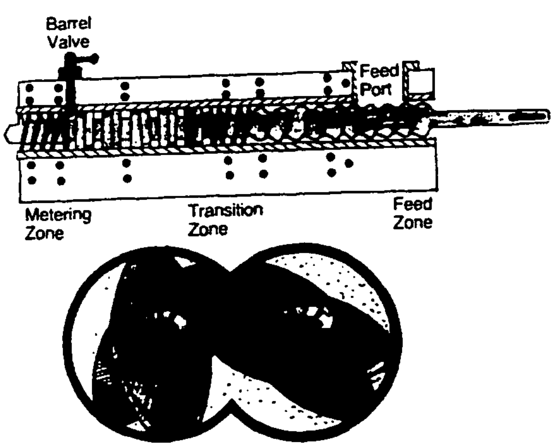

Fig. 11. Cross-section of a twin-screw extruder with barrel-valve (Baker Perkins).

Fig. 2). In counter-rotating, closely-intermeshing extruders the calender gap plays an important role. Material transported through this gap will experience high shear and elongational forces, thus giving high micro-mixing and dispersion. Since during normal operating conditions in counter-rotating extruders with reasonable calender gaps most of the material will pass this gap at least once it can be concluded that in counter-rotating extruders the average shear level is very low but a short-time, high-shear treatment is also present. Shear increase can be achieved by designing a conical housing and screw combination with a suitably-designed throttle zone as in the Cincinnati design or 'barrel-valve' as in the Baker Perkins design (Figs 10 and 11).

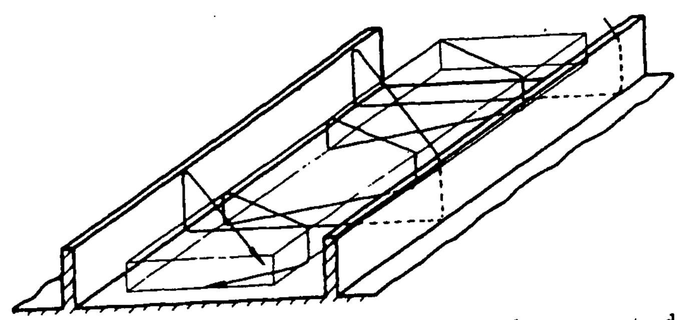

Fig. 12. Flow pattern in the channel of a single-screw extruder.

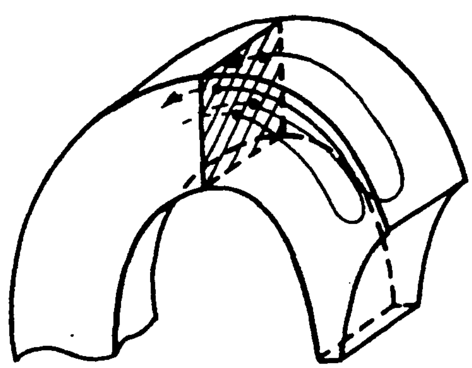

Fig. 13. Recirculation in the chamber of a twin-screw extruder.

MIXING AND HEAT TRANSFER

In an s.s.e. the flow field can be calculated easily from a judicious combination of cross-sectional and down-channel velocity profiles, to give a helical flow as indicated in Fig. 12. In this flow field the trajectory of a liquid particle is completely determined by the height at which

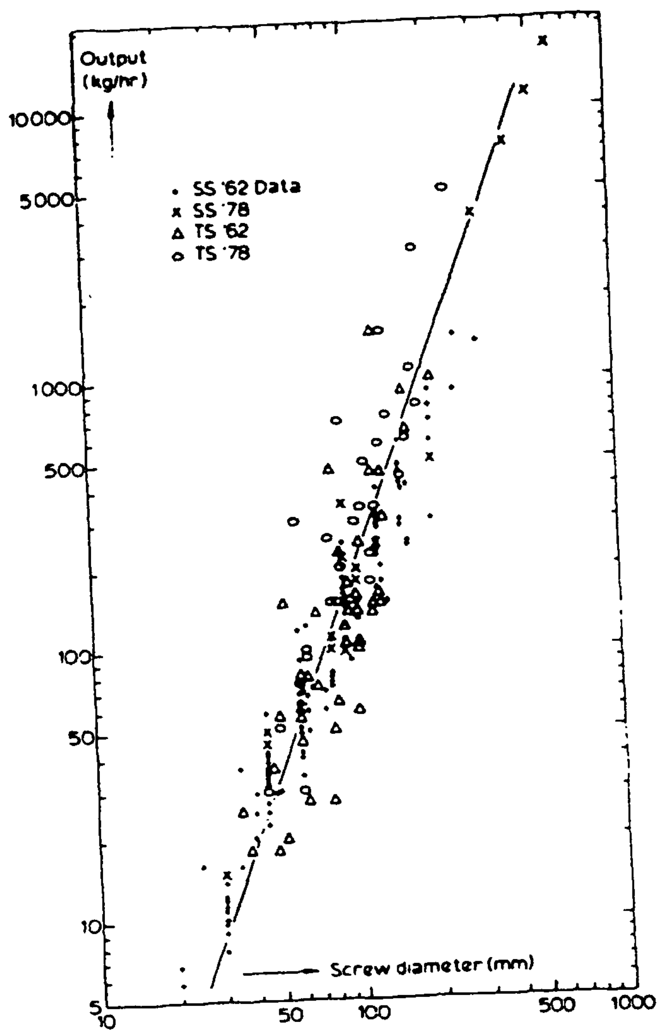

Fig. 14. Throughput of different extruder types according to supplier specifications.

it enters the channel in the fully-filled pumping zone. However, particles that enter this zone near the centre of the helical flow field will never approach the wall, and because of the poor thermal conductivity of some biopolymers heat generated by viscous dissipation in this region is only slowly removed by conduction to the wall. This can result in high temperatures along the middle of the channel.

Conclusion C: heat transfer

The situation in closely-intermeshing t.s.e.'s is different, as the material is redistributed near the intermeshing region. Material coming from the inner part of the helical flow field is transferred to the outer part near the wall (Fig. 13), thus ameliorating heat transfer. Since t.s.e.'s also have lower rotational speeds in general, with less energy dissipation, this ensures a good thermal homogeneity in the product.

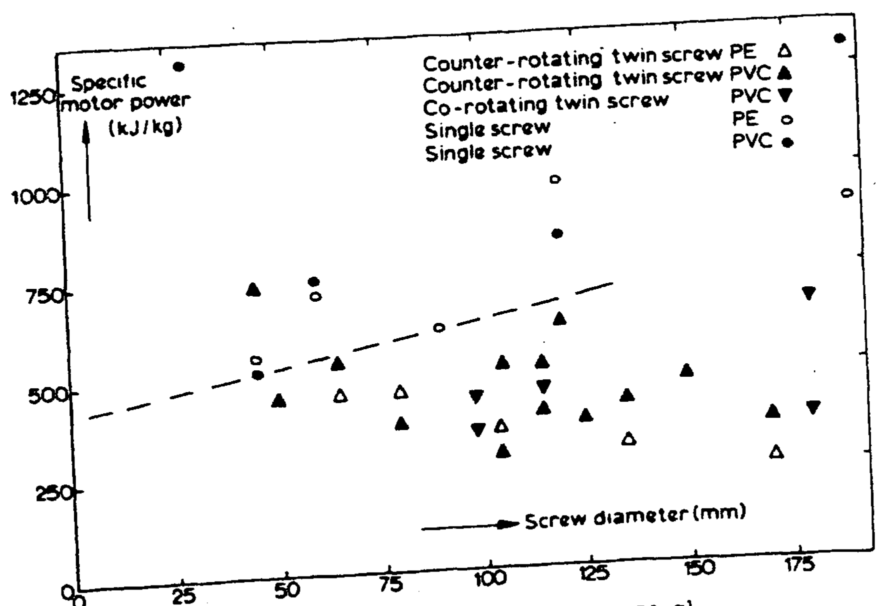

Fig. 15. Specific power consumption, kJ kg^{-1}.

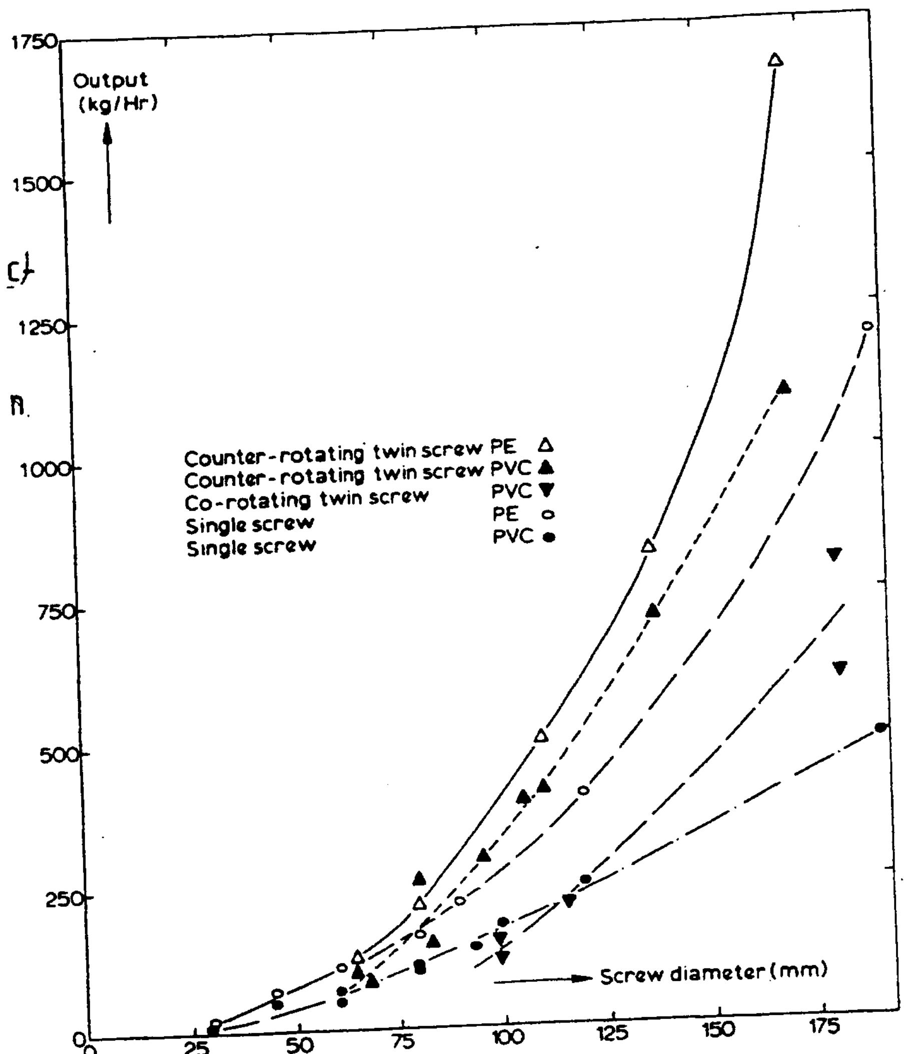

Fig. 16. Throughput as a function of screw diameter for single- and twin-screw extruders for 1962 and 1978.

AN ENGINEERING ANALYSIS

From the foregoing it will be clear that it is not easy to compare commercially-available twin- and single-screw extruders, although the screw diameter can provide a basis for this.

Unfortunately, not enough data are available on different kinds of food extruders to make a reliable and useful comparison. However, from our own experiments we have found enough similarities between the extrusion of some biopolymers and some synthetic polymers to justify a comparison with plastic machinery. Figure 14 compares the maximum output as specified by a number of machine manufacturers in 1962 and 1978 for single-screw and various types of twin-screw machines. This insignificant shift in the data between 1962 and 1978 is striking. Although there is a rough similarity between the output–screw diameter relationship for single-screw and twin-screw extruders, it can be seen that twin-screw extruders have in general a higher output than single-screw machines.

However, the t.s.e.'s run at much lower speeds (20–60 rpm) than the s.s.e.'s (100–400 rpm). This means that the effective transportation in twin-screw machines is much greater. The two screws almost double the uptake in the feed section and, moreover, under normal operating conditions there is no back pressure present in the feed zone. Regarding these differences in rotational speed and output, it is interesting to compare the specific motor power (motor power per unit of product) and throughput for the different types of machines (Figs 15 and 16). It can be seen that specific motor power is generally higher for s.s.e.'s than it is for t.s.e.'s. In Figs 17 and 18 the throughputs and specific power consumption are given for maize grits in a conical t.s.e. provided with a screw set suitable for PVC. The influence of die diameter and moisture content is shown in Figs 19 and 20.

It is possible to obtain identical viscograms from a conical t.s.e. and an s.s.e. (see Figs 21 and 22).

One of the specific advantages of t.s.e.'s is clear. An extruder is a thermodynamic unit. Most of the power to drive the screws is converted into heat, but because of losses in the motor unit, the gear system and the bearings, heat produced by viscous dissipation is more expensive than that acquired from simple heaters. The heat supplied through the barrel may even be waste-heat recovered from other equipment. As most polymers are susceptible to thermal degradation, good thermal control is needed. If the amount of heat produced by viscous dissipation is far more than that supplied through the barrel, temperature control by the barrel heating might become difficult so that the screw speed has to be changed. The associated changes in output can be especially troublesome when the extruder is feeding

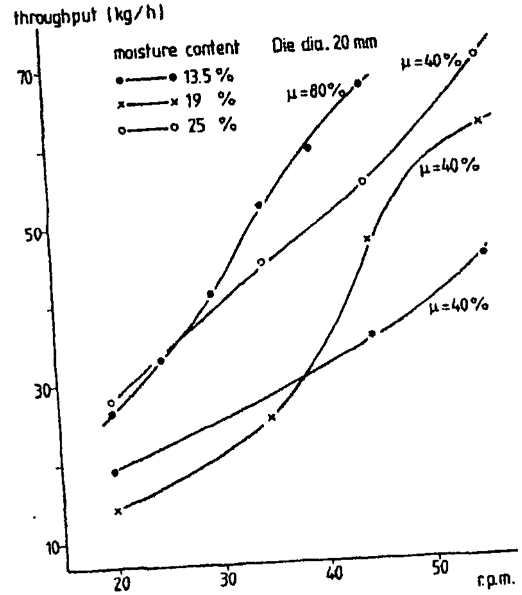

Fig. 17. Throughput related to rpm, moisture content and torque in a conical, twin-screw extruder.

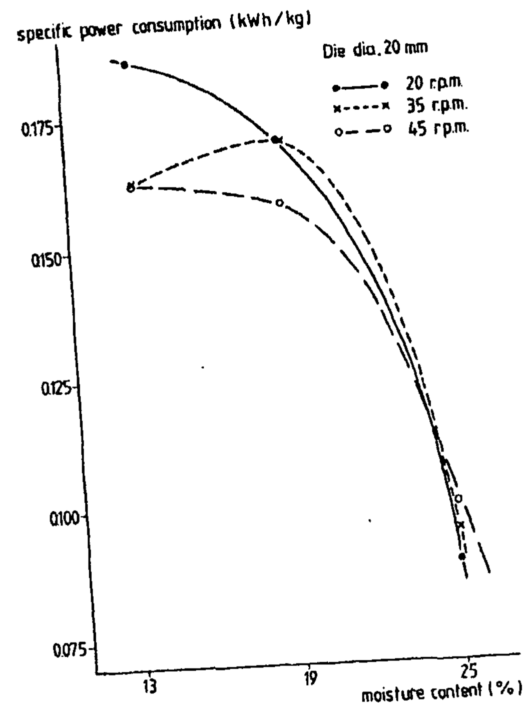

Fig. 18. Specific power consumption versus moisture content in a conical, twin-screw extruder, related to rpm.

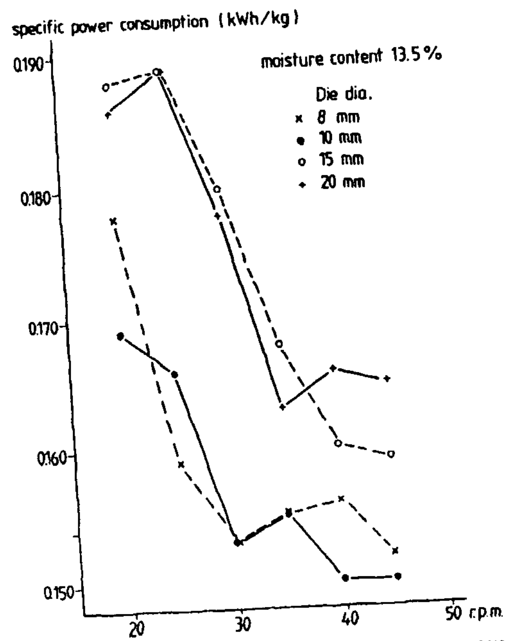

Fig. 19. Specific power consumption versus rpm, related to moisture content and die diameter.

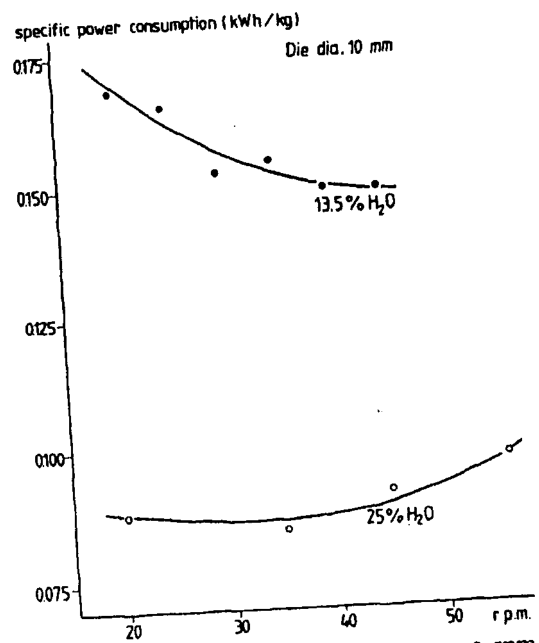

Fig. 20. Specific power consumption versus rpm related to moisture content for one die diameter.

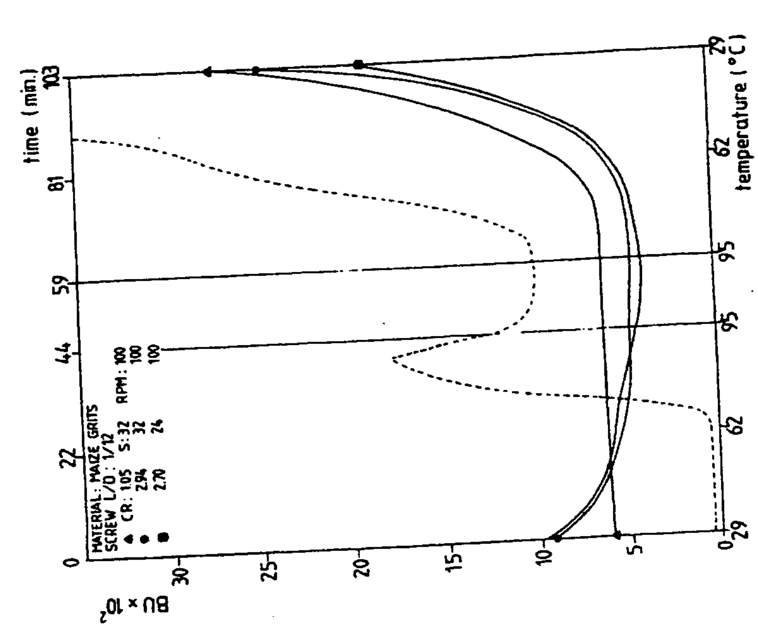

Fig. 21. Brabender viscogram for a single-screw extruder.

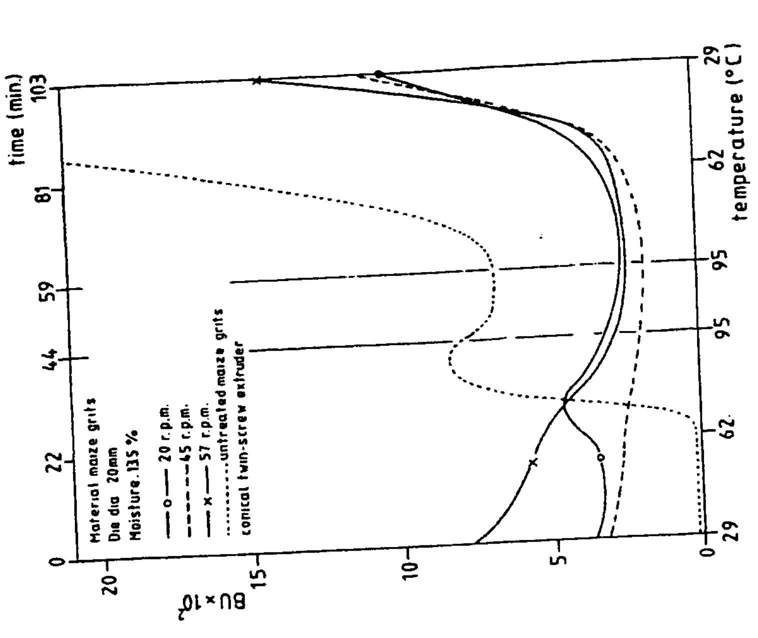

Fig. 22. Brabender viscogram for a co-rotating twin-screw extruder.

other machines. When the viscous dissipation is relatively small and the major part of the heat is supplied externally, as is the case in a t.s.e., then the screw speed and therefore the output can remain constant while the heaters can be used for controlling the process over a wide range of conditions. However, it should be realised that this is at the cost of the shear imposed on the material. The choice of a particular extruder type must be made by a judicious balance between shear requirements and other factors.

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Slip Flow in Partially Filled Screw Channel

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ABSTRACT: In polymer extrusion, occurrence of wall slip can considerably alter the flow and mixing behavior. In most cases, it occurs in partially filled screw channels because starved feeding is very common in industrial processing. In this paper an analytical model was developed to study the extrusion flow of Newtonian fluids with wall slip in a partially filled single screw extruder. Based on this analysis, the change of flow due to wall slip phenomenon was analyzed. The effects of percent channel fill, wall slip coefficient and viscosity on the flow and mixing are investigated.

INTRODUCTION

WALL SLIP PHENOMENA can occur in non-homogeneous flows such as capillary and extrusion flows of filled polymers due to the shear-induced migration of the filler particles. Occurrence of wall slip can considerably reduce the rate of deformation exerted on the polymeric fluid by 5 times or more [1] and alter the flow behavior. This can seriously affect the degree of mixing and even the entire processing behavior. Wall slip can also cause severe processing instabilities. Micro scale studies of slip phenomena in filled systems have often focused on filler particle migration due to the non-homogeneous flow field [2,3].

Previous theoretical work relating slip phenomenon in extrusion has been mainly devoted to fully filled channels [4–6] and complete numerical simulations are too complex for general industrial applications. In most cases, wall slip in extrusion occurs in partially filled screw channels because the use of

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starved feeding is very common. Experimental evidence has indicated that the flow and mixing behavior are significantly different in fully and partially filled systems. The percentage channel fill has been found to be one of key factors to dominate the mixing performance in an extruder [7]. In extrusion, the average residence time and throughput are very important parameters relating the productivity, mixing performance and degradation for temperature or shear sensitive polymers. These two parameters are mainly dominated by the down channel velocity. Although the cross channel velocity also influences the mixing behavior, it is not feasible to obtain the analytical solutions for cross channel velocities under this circumstance. The work in this paper is focused mainly on the down channel velocity in order to obtain comprehensive information for industrial applications.

An attempt has been made to model the extrusion flow of Newtonian fluids with wall slip in a partially filled single screw extruder. This model should form the basis to better understand the phenomenon of wall slip. The information from the predictions of this model also can provide guidance for industrial applications. This model in conjunction with mixing criteria can be further used to characterize the mixing performance subject to wall slip in partially filled extruders.

MODEL DEVELOPMENT

The flow geometry inside a screw channel is represented approximately by a rectangular channel with a top moving surface including slip on three surfaces and with a fluid-gas free surface. A sketch of this geometry is shown in Figure 1. Wall slip occurs at the barrel surface ($y = H$), the bottom screw surface ($y = 0$) and the

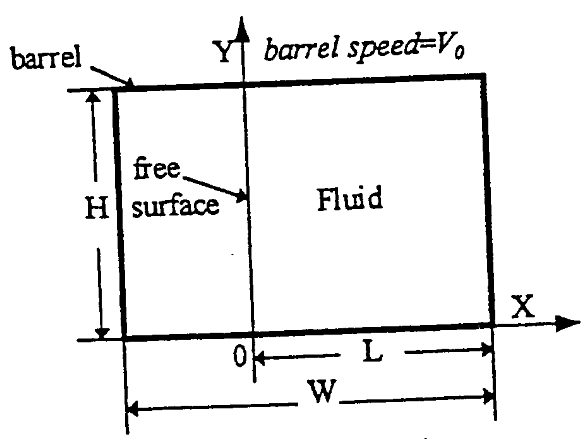

Figure 1. Sketch of flow geometry.

side wall of flight ($x = 0$). The slip velocity is represented by Navier's slip expression. There is a free surface between the liquid and gas ($x = L$). Basic assumptions are summarized as followed:

1. Newtonian Fluids
2. Steady state and creeping flow
3. Negligible flow through the flight gap
4. No pressure gradient in the down channel direction
5. Down channel velocity is a function of x and y, e.g., $V_z = V_z(x,y)$
6. Wall slip occurs on the barrel (top) surface, the surfaces of bottom screw and side left-hand flight wall.
7. Wall slip can be represented by Navier's slip equation ($t \cdot (V - V_s) = \beta nt{:}\Pi$); where t and n are the unit and tangent and outward normal vectors; V and V_s are the velocity and slip velocity; β is the slip coefficient and Π is the total stress tensor.
8. No vibration on the free surface.

The z-component of the equation of motion can then be written as:

$$\frac{\partial^2 V_z}{\partial x^2} + \frac{\partial^2 V_z}{\partial y^2} = 0 \tag{I}$$

The boundary conditions are listed as follows:

$$\left. \left(V_z + \beta_1 \mu \frac{\partial V_z}{\partial y} \right) \right|_{y=H} = V_0 \tag{1}$$

$$\left. \left(V_z - \beta_2 \mu \frac{\partial V_z}{\partial y} \right) \right|_{y=0} = 0 \tag{2}$$

$$\left. \left(V_z + \beta_2 \mu \frac{\partial V_z}{\partial x} \right) \right|_{x=L} = 0 \tag{3}$$

$$\left. \frac{\partial V_z}{\partial x} \right|_{x=0} = 0 \tag{4}$$

We can use the method of variable separation to solve the problem. Assuming that the z-component velocity, V_z, can be expressed as the product of a function of x and a function of y, e.g.:

$$V_z(x, y) = \Phi(x)\Psi(y) \tag{5}$$

where $\Phi(x)$ and $\Psi(y)$ are the x dependent function, y dependent function, respectively. Equation I can then be rearranged in terms of $\Phi(x)$ and $\Psi(y)$:

$$\frac{\Phi''(x)}{\Phi(x)} = -\frac{\Psi''(y)}{\Psi(y)} = -\lambda^2 \tag{6}$$

where λ is the eigenvalue to be determined.

Applying boundary conditions 3 and 4, we have an approximate expression for λ_n [7]:

$$\lambda_n = \frac{2n-1}{2L}\pi \tag{7}$$

Applying the boundary condition 2, the z-component velocity can be obtained:

$$\begin{aligned} V_z(x, y) &= \Phi(x)\Psi(y) \\ &= \sum_{n=1}^{\infty} k_n \cos(\lambda_n x)[\sinh(\lambda_n y) + \beta_2 \mu \lambda_n \cosh(\lambda_n y)] \end{aligned} \tag{8}$$

By using boundary conditions 1, we can obtain an expression for k_n,

$$\begin{aligned} \sum_{n=1}^{\infty} k_n \cos(\lambda_n x)[(1 + \beta_1\beta_2\mu^2\lambda_n^2)\sinh(\lambda_n H) \\ + (\beta_1 + \beta_2)\mu\lambda_n \cosh(\lambda_n H)] = V_0 \end{aligned} \tag{9}$$

where V_0 is the barrel velocity

$$V_0 = \pi ND \cos(\theta) \tag{10}$$

The values of k_n can be solved by using the property of orthogonality:

$$k_n = (-1)^{n+1} 4V_0 / (2n-1)\pi \,/\, (A + B) \tag{11}$$

where A and B are:

$$A = \sinh\left(\frac{(2n-1)\pi}{2}\frac{H}{L}\right)\left[1+\beta_1\beta_2\mu^2\left(\frac{(2n-1)\pi}{2L}\right)^2\right] \tag{12}$$

$$B = (\beta_1+\beta_2)\mu\frac{(2n-1)\pi}{2L}\cosh\left(\frac{(2n-1)\pi H}{2L}\right) \tag{13}$$

Finally, we can obtain an analytical solution for the down channel velocity:

$$V_z(x, y) = \sum_{n=1}^{\infty} k_n \cos\left(\frac{(2n-1)\pi}{2}\frac{x}{L}\right)\left[\sinh\left(\frac{(2n-1)\pi}{2}\frac{y}{L}\right) + \beta_2\mu\frac{(2n-1)\pi}{2L}\cosh\left(\frac{(2n-1)\pi}{2}\frac{y}{L}\right)\right] \tag{14}$$

The volumetric flow rate, Q, can be calculated as the following:

$$Q = \int_0^H \int_0^L V_z(x, y)dx\,dy \tag{15}$$

The all quantities we are interested in are non-dimensional for comparison. The dimensionless forms for the coordinates, x, and y, velocity, volumetric flow rate, Q and average time, t, are defined as:

$$x^* = x/L, \quad y^* = y/H \tag{16}$$

$$V_z^*(x, y) = V_z(x, y)/V_0 \tag{17}$$

$$Q^* = Q/V_0LH \tag{18}$$

$$t^* = t/(T/V_0) = HWV_0\,p/Q \tag{19}$$

where Q^* and t^* are the dimensionless volumetric flow rate and average time, respectively; T is the down channel length. In following sections, we will discuss the results.

RESULTS AND DISCUSSION

Effect of Navier's Slip Coefficient

Figure 2 shows the effect of Navier's slip coefficient on the volumetric throughput and average residence time. Note that, in this work, we assume the slip coefficients on the screw surfaces and barrel surfaces are the same. The dimensionless volumetric throughput decreases with increasing the values of slip coefficient while the average residence time increases. The average residence time increases almost three fold with a four fold increase of wall slip coefficient. This is induced by the substantial decrease of the barrel velocity, which in turn, reduces the average down channel velocity. The decrease of the flow rate under conditions with wall slip is directly related to the velocity distribution in the cross channel. The velocity contours at 50% channel fill are shown in Figures 3 and 4 under conditions with and without wall slip, respectively. The velocity under the condition with wall slip is considerably lower near top plate due to the wall slip, in comparison to that without slip. Although there exist certain values of the velocity at the bottom and right side wall for the case with slip, the decrease of the velocity near the top plate is the dominant factor in reducing the overall volumetric flow rate. It is also observed that the zone of low deformation under slip condition becomes more pronounced in lower right hand area.

The deformation rate of fluid under the condition of wall slip decreases substantially in comparison to the case of no slip at same channel fill and viscosity. The typical velocity gradient contours for both cases are shown in Figures 5 and 6. The

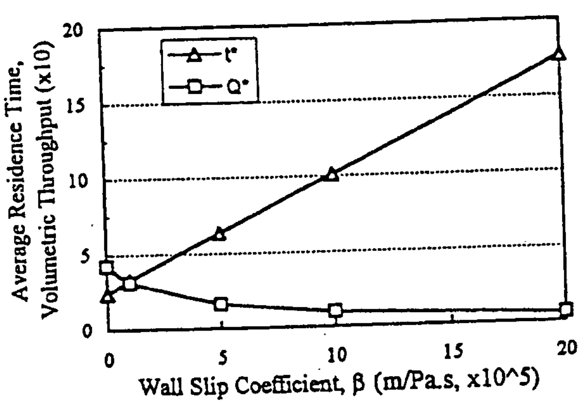

Figure 2. Effect of wall slip coefficient on the average residence time and flow rate (μ = 500 Pa·s; P = 50%).

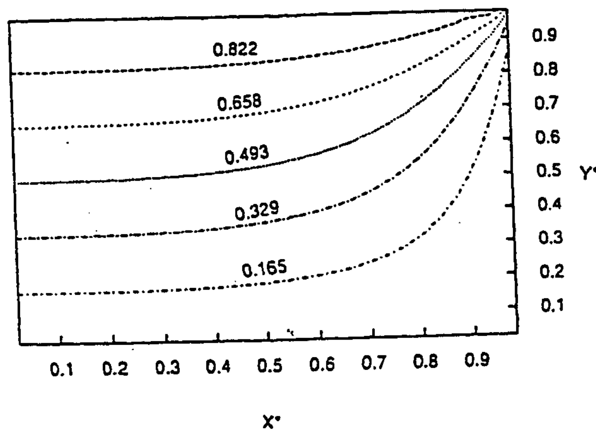

Figure 3. *Velocity profile under no slip condition (μ = 500 Pa·s; p = 50%).*

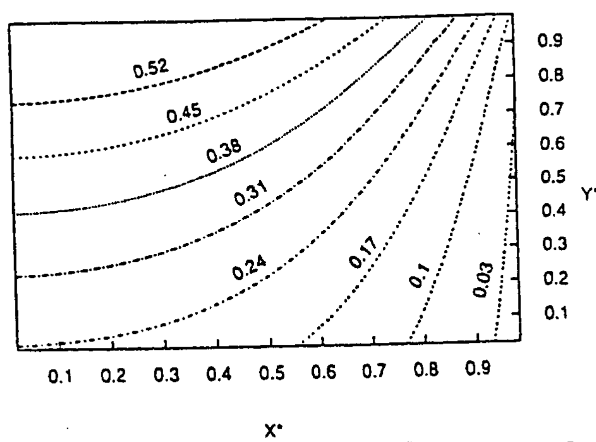

Figure 4. *Velocity profile under slip condition ($\beta=1\times10^{-5}$ m/Pa.s; μ = 500 Pa·s; p = 50%).*

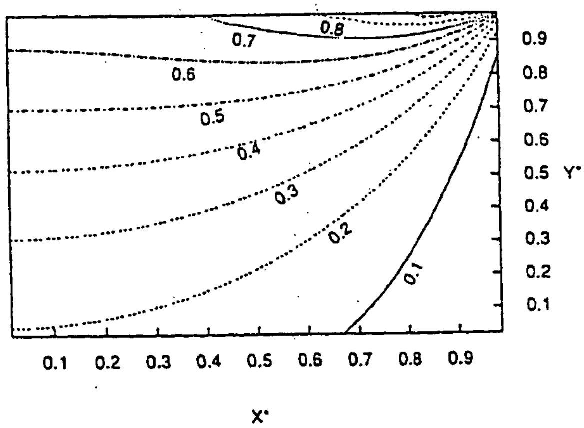

Figure 5. *Deformation rate ($\partial V_z/\partial y$) profile under slip condition ($\beta = 1 \times 10^{-5}$ m/Pa.s; $\mu = 500$ Pa·s; $p = 30\%$).*

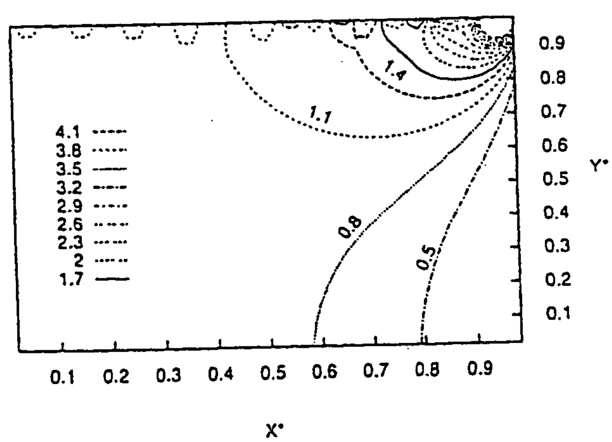

Figure 6. *Deformation rate ($\partial V_z/\partial y$) profile under no slip condition ($\mu = 500$ Pa·s; $p = 50\%$).*

velocity gradient ($\partial V_z/\partial y$) decreases by five fold under slip condition. Drop in deformation rate of $\partial V_z/\partial x$ is also observed under the slip condition (not shown in this paper). A similar trend of decreasing volumetric flow rate with increasing slip velocity is also obtained from the 1-D model [10]. On the other hand, the momentum flux also decreases. Although we only consider Newtonian fluids in this model, it can be expected that the non-Newtonian behavior will further reduce the rates of deformation.

For industrial processing, one way to increase the throughput at a given RPM is to increase the roughness of the wall of both barrel and screw surfaces. Although, the balance between the throughput and the extruder torque loading needs to be considered.

Effect of Percentage Channel Fill

The effect of the percentage channel fill on the dimensionless average residence time and volumetric throughput is shown in Figure 7 for a viscosity of 500 (Pa·s) and Navier's slip coefficient of 10^{-5} m/Pa·s. The volumetric flow rate, Q*, increases with increasing percentage channel fill for cases both with and without wall slip. This is due to the increase of average down channel velocity caused by increase of channel fill. At a given percent channel fill, the flow rate under conditions of wall slip is lower than without wall slip. It is also noticed from Figure 2 that the difference of the average residence times between no slip and slip conditions becomes smaller with increasing channel fill. So does for throughput.

The percentage channel fill is directly related to the percentage drag flow in absence of pressure flow. The percentage drag flow has been found experimentally to have an important role on the mixing performance in extruder [4,9]. Under wall slip conditions, reducing the percentage channel fill will result in the decrease of deformation rates. As seen in Figures 5 and 8, the deformation rate can be reduced considerably by only decreasing percent channel fill from 50% to 30%. This indicates that with the occurrence of wall slip, the percentage channel fill may severely reduce the mixing performance in an extruder by decreasing the deformation rates.

Effect of Viscosity

The effect of viscosity on the volumetric throughput and average residence time is shown in Figure 9 for a 50% filled screw channel. For high viscosity, low Reynolds number and creeping flow, the dimensional throughput and average residence time without wall slip are independent of viscosity in a partially filled screw channel (i.e., absence of pressure flow) at given barrel speed since the down channel velocity is independent of viscosity for Newtonian fluids. However, the throughput under wall slip conditions decreases considerably with increasing viscosity, while the residence time decreases. Increasing viscosity will increase the

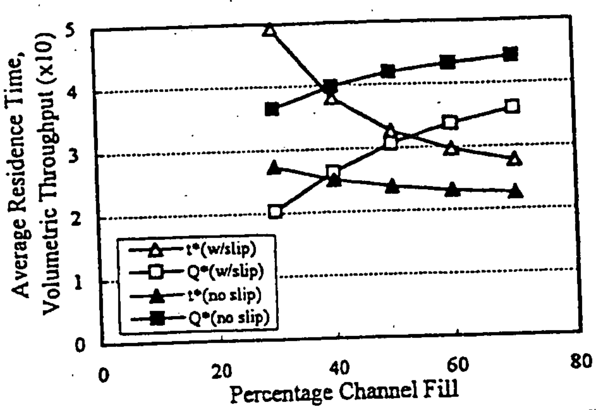

Figure 7. *Effect of percentage channel fill on the average residence time and flow rate (β=1 × 10^{-5} m/Pa.s; μ = 500 Pa·s).*

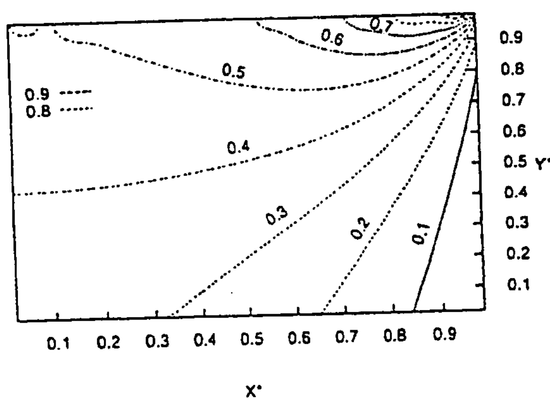

Figure 8. *Deformation rate ($\partial V_z/\partial y$) profile under slip condition (β=1 × 10^{-5} m/Pa.s; μ = 500 Pa·s; p = 50%).*

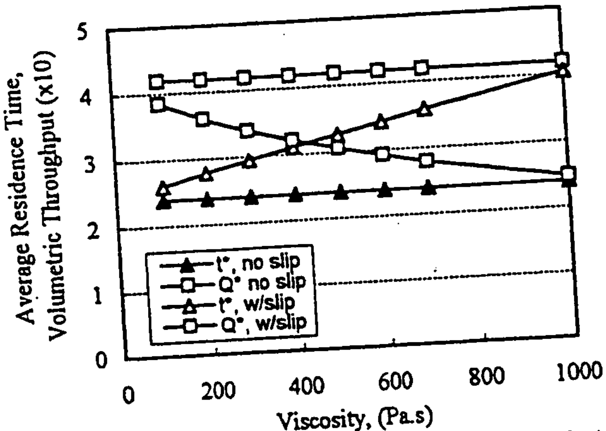

Figure 9. Effect of viscosity on the average residence time and flow rate ($\beta=1\times10^{-5}$ m/Pa.s; p = 50%).

shear stress on barrel surfaces at a given RPM. The wall slip velocity at barrel surface then decreases since the slip velocity is proportional to the wall shear stresses (see the model development assumption No. 7). The actual velocity of the fluid in down channel direction then decreases with increasing viscosity.

On the other hand, the rates of deformation are also reduced with increasing viscosity. Two contours of velocity gradients, $\partial V_z/\partial y$, at the same channel fill and slip coefficient are shown in Figures 8 and 10 for two fluids with viscosity values of 500 and 100 Pa · s, respectively. The rate of deformation decreases by

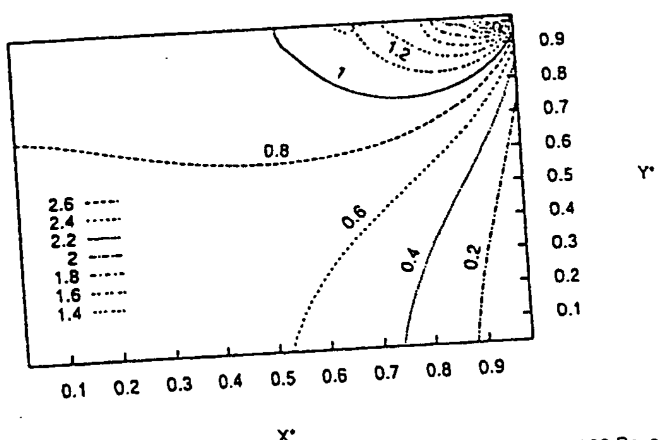

Figure 10. Deformation rate ($\partial V_z/\partial y$) profile under slip condition ($\mu=100$ Pa · s; $\beta=1\times10^{-5}$ m/Pa.s; p = 50%).

two with an increase in viscosity from 100 to 500 Pa · s. Note that the low deformation regime in the lower right hand side becomes more pronounced under slip conditions.

For industrial applications, reducing the values of the viscosity becomes more important under condition of slip. Decreasing viscosity (for example, by raising the barrel temperature) can increase the throughput and reduce the average residence time at a given RPM. It also improves the distributive mixing performance by increasing the rates of deformation and by reducing stagnation zones. However, the resultant lower stress levels may have opposite effect on the dispersive mixing. Therefore, the issuer of viscosity needs to be considered for each application of specific system.

CONCLUSIONS

Based on the analytical model developed we conclude that:

- The dimensionless volumetric throughput under condition of wall slip decreases in comparison with the no slip cases, while the average residence time increases. The rates of deformation also are reduced considerably under slip conditions.
- The throughput and rates of deformation decrease with increasing slip coefficient and viscosity. The average residence time increases with increasing wall slip coefficient.
- Increase on the percent channel fill will increase the throughput and deformation rates. Increasing the percent channel fill gives rise to a decrease of the average residence time.

NOTATION

H height of screw channel, m
k_n coefficient
L width of fluid, m
n unit normal vector
p percent channel fill
Q^* dimensionless volumetric flow rate
T down length of screw, m
t unit tangential vector
t^* dimensionless average residence time
V^* dimensionless velocity
V_0 barrel velocity, m/s
W width of screw channel, m
x^* dimensionless x
y^* dimensionless y

Greek

β_1, β_2 Navier's slip coefficient, *mPa/s*
λ eigenvalue
μ viscosity, $Pa \cdot s$
Π total stress tensor

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